Int. J. Multiphase Flow Vol. 10, No. 5, pp. 635–641, 1984
Printed in the U.S.A.

0301–9322/84 $3.00 + .00
Pergamon/Elsevier

BRIEF COMMUNICATION

ANNULAR BUOYANT JETS

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(*Received* 23 *December* 1982; *in revised form* 10 *October* 1983)

Thin sheets of flowing water or annular water jets describe various shapes under the action of surface tension, gravity and curvilinear motion. Taylor (1959) pointed out that this has been known since Savart (1833). If the jet converges, it encloses a bubble of air, as is commonly observed under low discharge conditions with some domestic taps. The enclosed air can be at a different pressure from that outside the bubble and Hopwood (1952), Lance & Perry (1953), who formulated a theoretical description, and Taylor (1959) showed that the bubble can take on a variety of beautiful shapes when the pressure is controlled.

As far as the writer is aware it has gone unnoticed that similar events occur if an annular jet is discharged into a miscible fluid. However, several differences arise. Firstly, the absence of surface tension prevents the arbitrary control of pressure unless a continuous supply of fluid is added to the interior of the bubble. Secondly, the jets will entrain fluid from both sides and, in consequence, a recirculating flow, which cannot be ignored, is created within the bubble. Lastly, unless fluid is added to the bubble, the contents of the bubble will be composed of the same material as that of the jet. Therefore, if the jet fluid is of different but comparable density to that of the ambient fluid, the pressure difference between the inside and outside of the bubble will vary significantly with elevation. For this reason, the title of this communication refers to buoyant annular jets.

The particular work to be described was carried out with respect to an industrial example, in which a hot annular jet of air is discharged upwards into colder air. The annulus is formed between the wall of a vertical cylindrical tube and cylindrical cannisters of spent nuclear fuel, which the air is cooling. This is part of a proposed scheme for the storage of such fuel for periods of about 100 years, (Bradley & Brown 1982).

THE EXPERIMENT AND PHOTOGRAPHIC RESULTS

The experiments were carried out with an annular jet of dyed water discharging vertically upwards into either water or into a 24% solution of common salt in water, when the density difference between the two fluids was 180 kg/m^3. In the latter case, the correct values of Reynolds and Richardson number applicable to the prototype could be obtained simultaneously with relatively small experimental equipment.

The ambient fluid was contained in a cubical Perspex tank with internal sides of length 305 mm. The jet, which was also made of Perspex, was set in the centre of the bottom of the tank with its discharge flush with the bottom surface. The outer diameter of the annulus was 57 mm and its width was either 1.27 mm or 3.81 mm. The annulus was 110 mm long and care was taken to ensure that it was uniformly supplied with water.

The top of the tank was sealed, except for numerous small holes around its edge to allow uniform discharge of fluid to an upper open tank with an overflow.

The tank was filled with the desired ambient fluid, except that a 12 mm layer of dyed water was layered on top of salt solutions, since this seemed to effect a smoother start to experiments. Before an experimental run was started, the annular jet and all lines leading to it were filled with dyed water. During a run, dyed water was pumped from a storage tank through a rotameter.

An experiment simply consisted of setting the required flow abruptly and running until the view through the Perspex tank was obscured by dye when the ambient fluid was water or until the supernatant layer of dyed saline solution, which formed when the ambient fluid was a salt solution, approached the bottom of the Perspex tank. The interface between the supernatant layer and the original salt solution was clearly defined and no dye penetrated into the lower layer.

Jet flowrates of from $1.26 \times 10^{-4}\,m^3/s$ to $6.31 \times 10^{-4}\,m^3/s$ with increments of $1.26 \times 10^{-4}\,m^3/s$ were employed. Reynolds numbers based upon a linear dimension of twice the annulus width were therefore varied from 1440 to 7200. Experimental runs lasted less than 30 s and photographs of the contents of the experimental tank were taken every 1.22 s.

The form of the jet was independent of the flowrate or the annulus dimensions when water was used as the ambient fluid and figure 1 shows some photographic results. Figure 1(a and c) are the first photographs from one run and figure 1(b) is the first photograph from another run. Figure 1(a) shows that the jet was initially cylindrical but rapidly collapsed to the closed bubble forms of figure 1(b and c). Figure 1(b) clearly shows that the jet formed the surface of the bubble. Figure 1(c) shows that the contents of the bubble were rapidly entrained and changed to substantially that of the jet fluid, though some ambient fluid may be continually entrained by the jet and passed to the bubble's interior. The interior of the bubble is seen to be obscured before the plume rising from the top of the bubble reached the top of the Perspex tank.

It is emphasized that the height and shape of the bubble did not vary with the jet flowrate or dimensions.

No photographs exhibiting a cylindrical jet form, like that of figure 1(a), were obtained when using the 24% salt solution as the ambient fluid, though photographs similar to those of figure 1(b) were obtained. Figure 2 shows photographs of well-established bubbles using salt solutions with a jet annulus width of 3.81 mm and flowrates of $1.26 \times 10^{-4}\,m^3/s$ and $3.98 \times 10^{-4}\,m^3/s$. These clearly show that the height of the bubble increased with the flowrate, as the influence of buoyancy decreased compared to that of jet momentum. The height of the bubble approached that observed when water was the ambient fluid if the flowrate was increased to $6.31 \times 10^{-4}\,m^3/s$ and approached even closer for the annulus width of 1.27 mm.

It is interesting to note the ring-like wave structure on the surface of the bubble in figure 2. These waves detach as billows before the jet finally converges to form an essentially cylindrical plume. Figure 2 may give a false picture of the proportion of jet fluid carried forward to form this plume since the dye in the billows obscures observations of the plume.

THEORETICAL INTERPRETATION

Figure 3 illustrates the theoretical model. In particular, the angle at which the jet converges on the axis of symmetry is illustrative of the angle finally calculated from the theory for jets of water into water. The angle of the jet to the axis tends to 90° as the bubble height reduces and as the influence of buoyancy increases.

The radial position of the jet's centreline is initially x_0 and is generally x. The height of the jet's centreline is y, the jet thickness is initially t_0 and is generally t.

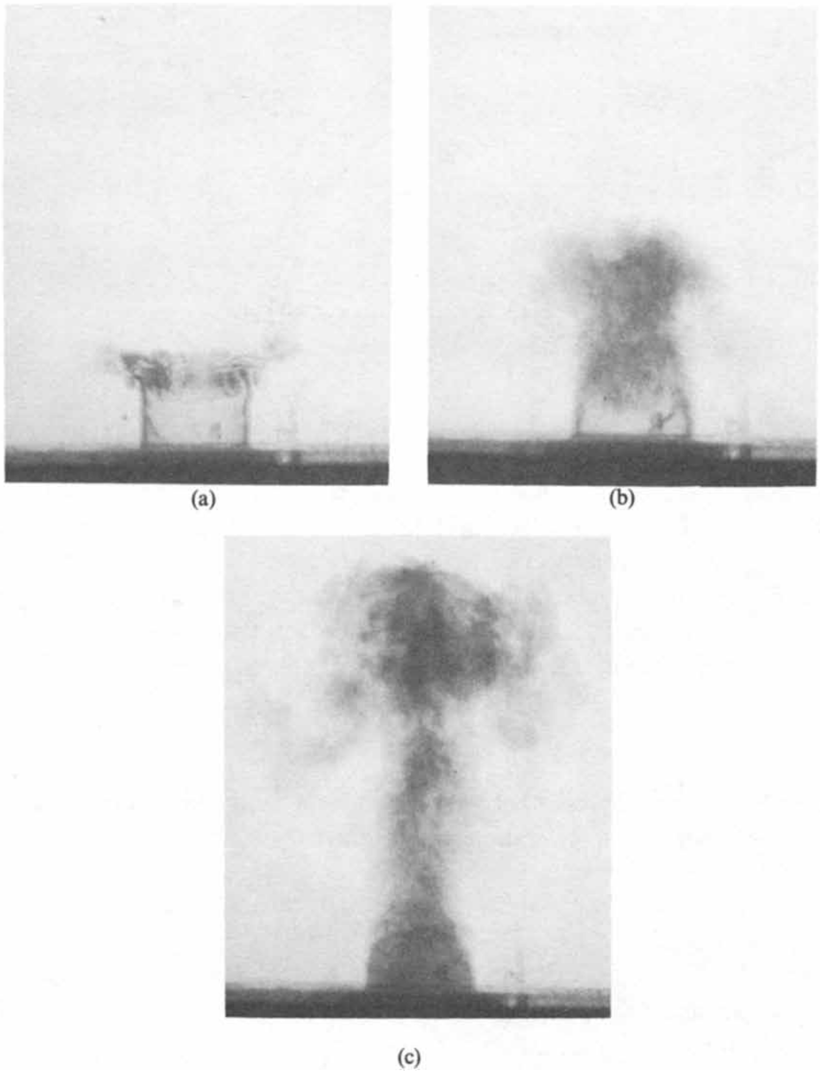

Figure 1. Early stages of discharge of an annular jet of water into water. (a) 200 gal/hr; (b) 400 gal/hr; (c) 200 gal/hr.

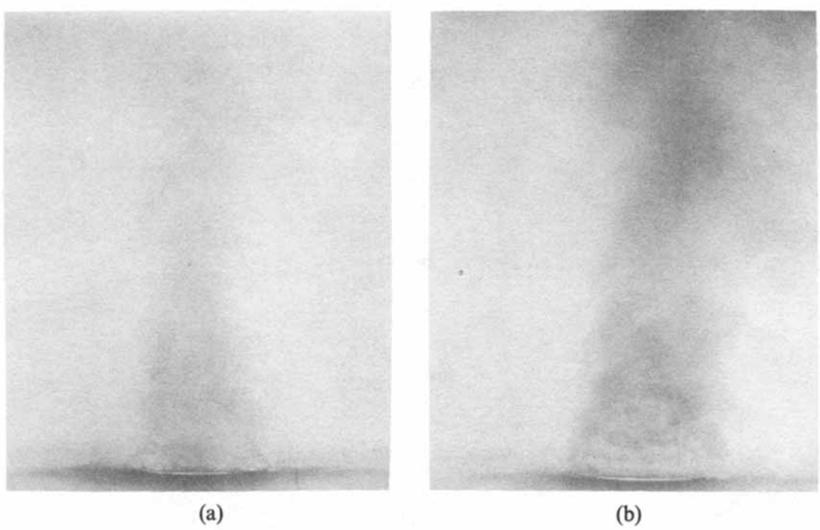

Figure 2. Discharge of water from largest annulus into 24% salt solution. (a) 100 gal/hr; (b) 300 gal/hr.

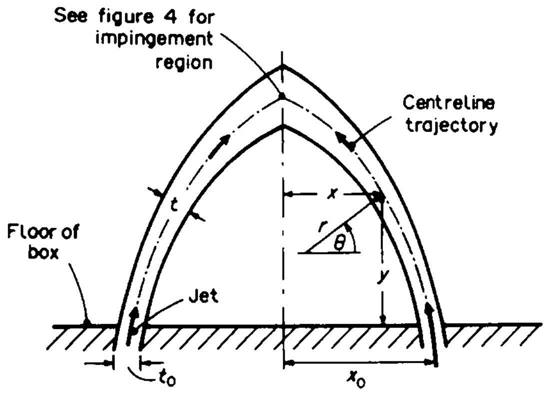

Figure 3. Dimensions of system analysed.

The assumed jet velocity profile is of the so-called "top hat" form and is thus uniformly U. Initially it is U_0. The radius of jet curvature is r and the normal to the jet's centreline extends an angle of θ with respect to the horizontal.

Let the pressure in the ambient fluid be Δp_0 above that inside the bubble at $y = 0$. On the assumptions that $(t \sin \theta)$ is small compared to y and that the fluid inside the bubble is the same as that of the jet, the pressure difference between the inside and outside of the bubble is

$$\Delta p = \Delta p_0 - \Delta \rho g y \tag{1}$$

where $\Delta \rho$ is the density difference between the ambient fluid and that of the jet, which is inside the bubble, and g is acceleration due to gravity.

The curvilinear motion also gives

$$\Delta p = \rho \frac{U^2 t}{r} \tag{2}$$

where ρ is the density of the jet fluid, which, with little loss of accuracy, can be set equal to that of the jet upon discharge.

The momentum of the jet in the direction of flow is conserved. Thus

$$U^2 t x = U_0^2 t_0 x_0. \tag{3}$$

Combining [1]–[3] yields

$$\frac{1}{R} = \frac{X}{N}(P - Y) \tag{4}$$

where

$$X = \frac{x}{x_0}, \quad Y = \frac{y}{x_0}, \quad R = \frac{r}{x_0} \tag{5}$$

$$P = \frac{\Delta p_0}{\Delta \rho g x_0} \tag{6}$$

$$N = \frac{m U_0}{2\pi \Delta \rho g x_0^3} \tag{7}$$

and where m is the jet's mass flowrate upon discharge.

We also have

$$\frac{1}{R} = \frac{d\theta}{dS} \qquad [8]$$

$$dX = -dS \sin\theta \qquad [9]$$

$$dY = dS \cos\theta \qquad [10]$$

where

$$S = \frac{s}{x_0} \qquad [11]$$

and s is the length of the jet's trajectory.

Using [4] and an assumed value for P, [8]–[10] are integrated to determine the coordinates of the jet's trajectory and the reduced pressure difference $(P - Y)_a$ at the axis of symmetry. However, [4] cannot be employed for the special case of $N = \infty$, when $\Delta\rho = 0$. Applying this condition to [1], it is found that the term Y must be omitted from [4] and the constants P and N can be grouped to form a new constant independent of $\Delta\rho$. The required integrations are then carried out.

To proceed further, we refer to the model of events near the axis of symmetry illustrated in figure 4. The jet flow is shown to divide into a cylindrical jet rising above the bubble and a recirculation flow, which is rather less than that of the cylindrical jet. The stagnation pressure of the stagnation stream time is clearly an important parameter in relation to the pressure difference over the jet approaching the axis of symmetry and the hypothesis is put forward that the ratio of these two parameters would not vary substantially with bubble height. A more detailed examination of events near the axis of symmetry cannot be justified by the experimental results available. In addition it is noted that an estimate of the stagnation pressure, even of the jet's centre streamline, cannot be made at this time. Instead, recourse is made to the theory and experimental evidence concerning plane jets, as presented by Schlichting (1955), which shows that

$$\frac{p_s}{\Delta\rho g x_0} = 2.88\,\frac{N}{S} \qquad [12]$$

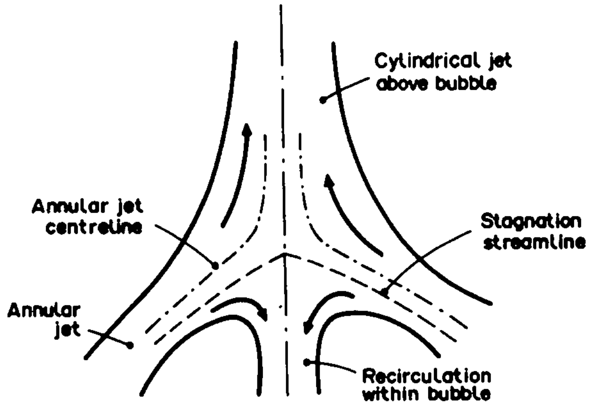

Figure 4. Conception of annular jet impingement on axis of symmetry.

where p_s is the centreline stagnation pressure. The ratio of Δp at the axis of symmetry from [1] to p_s from [12] therefore gives the final statement of the hypothesis as being that the parameter

$$M = \frac{S_a(P - Y)_a}{N} \qquad [13]$$

is a constant independent of bubble height, where S_a is the value of S at the axis of symmetry. A value of P must be chosen in performing the integration noted above and then values of M and Y_a will be determined. If the value of M is not the chosen one, P must be adjusted until the chosen one is obtained together with the corresponding value of Y_a.

Measurements of the bubble height were taken from the photographs. It is evident from figures 1 and 2 that considerable judgement had to be exercised since the tops of the bubbles tended to be obscured. Some photographs had to be rejected for this purpose but as many measurements as possible were made from the sequence of photographs for each run and the average values are plotted in figure 5 in terms of Y_a vs $N^{-1/3}$. Results for the two annulus widths agree satisfactorily with each other.

The theoretical line with $M = 1.5$ was chosen to agree with the value of $Y_a = 2$ for non-buoyant jets ($N = \infty$) and this line also follows the trend exhibited by buoyant jets. Little improvement is obtained with respect to the buoyant jets by choosing $M = 0.75$ but the prediction for non-buoyant jets changes substantially. It may be noted that the assumption of the constant of proportionality in [12] for a plane jet yields a value of M close to 1.5.

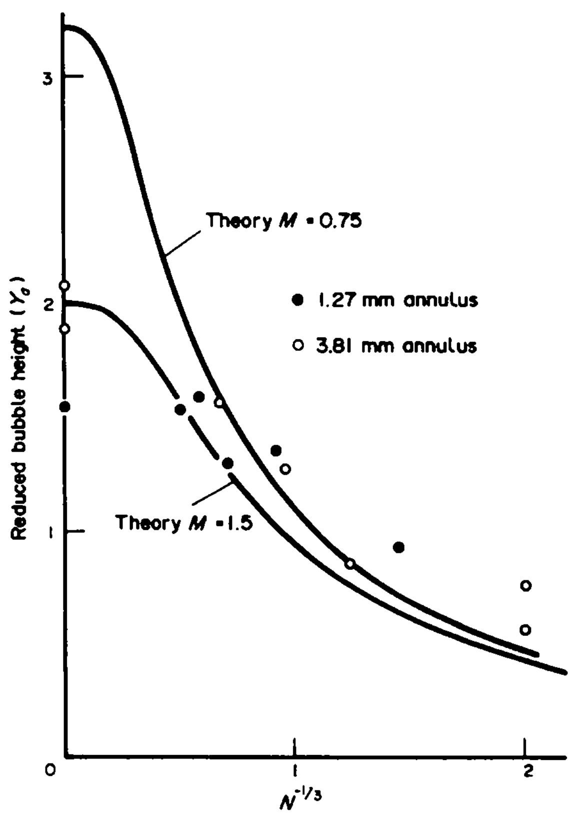

Figure 5. Variation of bubble height with buoyancy parameter N.

It must be stressed that the theoretical interpretation given above chiefly shows that the observed phenomenon is explicable. At the lowest flowrate but not at any others, the jet has a Reynolds number that indicates that it would be laminar upon discharge, though it would subsequently become turbulent. Thus for this flowrate, or for the largest value of $N^{-1/3}$ for each jet width in figure 5, a smaller value of M might be appropriate. It is also noted that any entrainment of ambient fluid by the jet to be passed to the bubble's interior has been ignored in the theory. This would lower the density difference and, in consequence, smaller values of $N^{-1/3}$ should be associated with the data points in figure 5. This would tend to give rather better agreement between theory and experiment but a substantial experimental program is needed before a more detailed analysis could be justified.

Acknowledgements—The authors are grateful for the help of H. S. Oates and G. E. Green in the construction of the apparatus.

The work was carried out at the Central Electricity Generating Laboratories and is published by permission of the Central Electricity Generating Board.

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